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BRL

REPORT NO. 867
MAY 1953

A MATHEMATICAL FORMULATION FOR ORDVAC COMPUTATION OF THE PROBABILITY OF KILL OF AN AIRPLANE BY A MISSILE

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ABERDEEN PROVING GROUND, MARYLAND

ERRATA SHEET

For

BRL REPORT NO. 867

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Page 8 - Line 20 Replace (x,y,x) by (x,y,z).

" 10 - Label the diagram "FIGURE 1".

" 11 - Line 10, 15 Insert minus signs in front of $K_1(x)$ and $K_2(x)$

" 12 - Line 5 Replace P by E

" 14 - Line 9 from the bottom Replace than by then

" 14 - Line 8 from the bottom Replace A by B

" 14 - Last Line First expression in brackets should be:

$$(P - R_k)\ A_k (P - R_k)^T - 1.$$

" 17 - Replace Figure 2 by Figure 1 of this note.

" 20 - Line 4 from the bottom - Replace $\left(\frac{r}{L\ 2\pi}\right)^3$ by $\left(\frac{r}{L}\sqrt{\frac{2}{r}}\right)^3$

" 21 - Line 9 Replace $t_{\gamma - L}$ by $- t_{L - \gamma}$.

" 21 - Line 4 Replace $\frac{1}{\ }$ by $\sqrt{\frac{1}{\ }}$

" 22 - Line

$$\frac{1 \quad 3}{\sigma_3} \quad \left(\frac{1 \quad 3}{\sigma_3}\right)^2.$$

" 23 - Line 12 Insert "times" between 8 and the

" 27 - a. Insert NO on negative exit from descrimination box Is $C_{i,j}$ in spray region?. (middle left hand side of figure).

b. Insert $<$ between i and I_j in discrimination Is $i < I_j$? (lower left area of figure).

c. Insert ? at end of discrimination box ending

... for a kill. (bottom of middle of figure).

Page 33 Line 5 from the bottom Replace $\frac{1}{2\pi}$ by $\sqrt{\frac{1}{2\pi}}$.

Page 34 Add a paragraph at the end. This latter procedure for obtaining normally distributed numbers from uniformly distributed ones was coded in a later revision of the problem. It used a sum of 5 or 6 uniformly distributed numbers properly normalized and with a small correction to improve the tails of the distribution. The time saving in the running of the problem amounted to about 30% to 40% of the previous time. This is due to the elimination of the square root and logarithm routines needed in the former procedure.

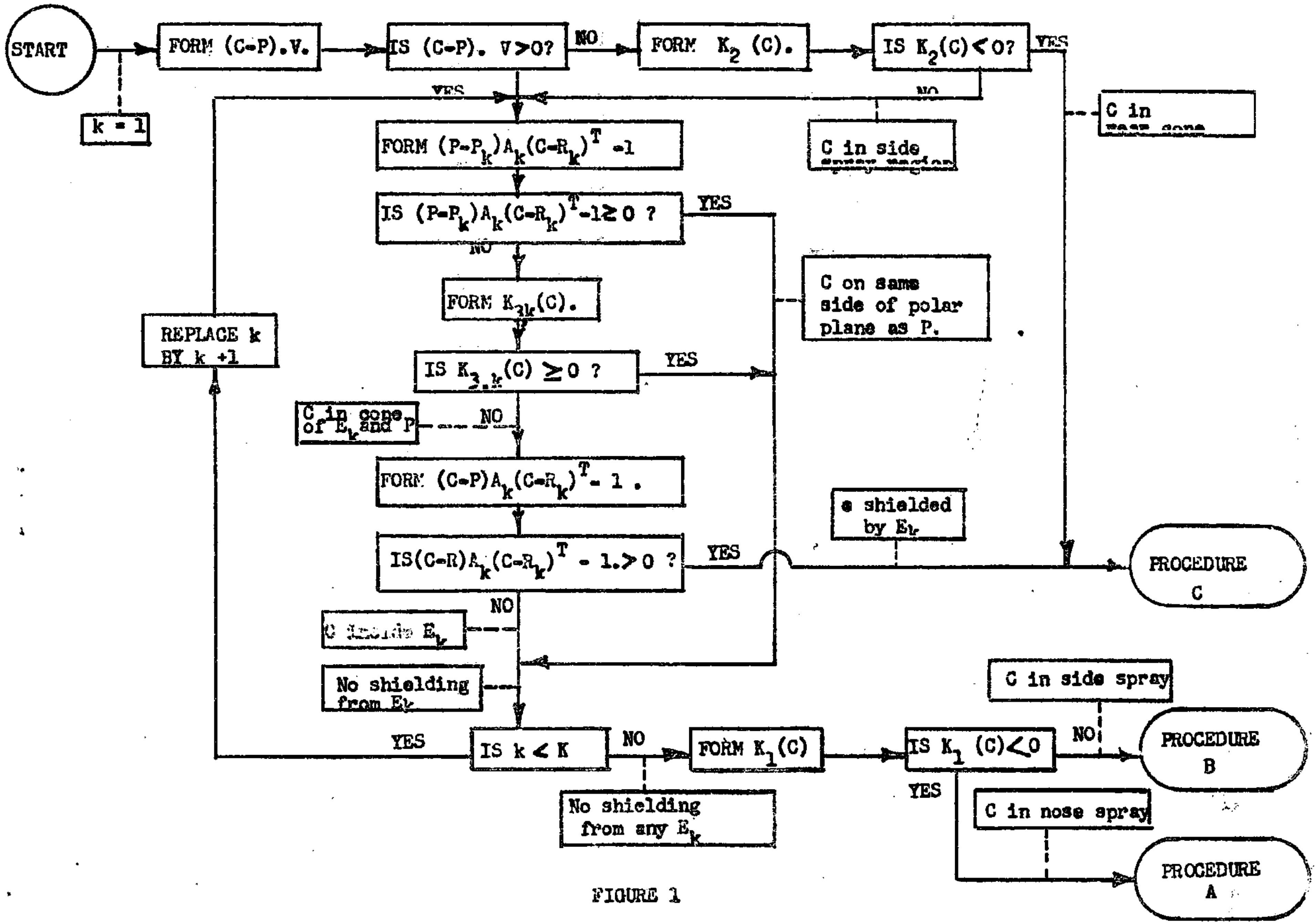

FIGURE 1

CORRECTED VERSION OF FIGURE 2 OF BRL REPORT NO. 867

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REPORT NO. 867

May 1953

(All previous changes have been inserted as of October 1964.)

A MATHEMATICAL FORMULATION FOR ORDVAC COMPUTATION OF THE PROBABILITY OF KILL OF AN AIRPLANE BY A MISSILE

M. L. Juncosa

D. M. Young

Project No. TB3-0138 of the Research and
Development Division, Ordnance Corps

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 867

MLJuncosa/DMYoung/lbe
Aberdeen Proving Ground, Md.
May 1953

A MATHEMATICAL FORMULATION FOR ORDVAC COMPUTATION OF THE PROBABILITY OF KILL OF AN AIRPLANE BY A MISSILE

ABSTRACT

In Ballistic Research Laboratories Memorandum Report No. 530, "Lotto Method of Computing Kill Probability of Large Warheads", F. G. King describes a random sampling method for determining kill probabilities of a large warhead against an airplane. To obtain the results the method requires a physical model, hand drawing of random numbers, and the use of kill probability curves for the vulnerable components of the airplane.

In this report a mathematical model for the purpose of solving the problem on a high-speed digital computing machine is presented. This model is based on J. von Neumann's suggestion that the airplane be replaced by several ellipsoids resembling the fuselage, wings, and engines. The necessary formulas for computation are derived from the basic geometric model.

The kill probabilities are determined by three-dimensional integrals which are evaluated either by random sampling methods or by straightforward numerical quadratures. These methods are compared from the viewpoints of accuracy, speed, and machine storage requirements. Limited comparison of the results and some remarks about applicability of more general problems are also made.

The method of generation of the pseudo-random numbers used in the random sampling procedures is also described in the appendix.

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I. INTRODUCTION

I.1 Description of the Problem

An airplane may be brought down by a bursting warhead by damage to its structure by blast or by destruction by fragments of the warhead casing of a sufficient number of vital components, which may include pilots, without which the airplane could not maintain controlled flight.

When the criteria for a kill for a particular airplane are established, using basic laws for combining probabilities, theoretically one could get the probability of obtaining a kill on the airplane with a particular warhead from a knowledge of the following:

a) the joint probability distribution of the burst point of the warhead, and the velocity (speed and direction) of the missile at burst point,

b) the velocity of the airplane at the time of burst,

c) the probability of destruction of the structure by blast as a function of directed distance of the missile from the airplane, relative velocities of the missile and airplane, wind and air density at time of burst,

d) the static distributions of fragment size, shape, weight, and velocity at time of burst,

e) the aerodynamic drag on the fragments as a function of the size and shape of the fragments and of the air structure at the time of burst,

f) the probability distribution of the air structure, ?

g) the location of vital components and of shielding components on the airplane, and

h) the probability of destruction of each vital component as a function of direction, size, shape, and striking energies of fragments.

Obviously, for practical considerations, such as the impossibility of getting some of the above desired probability distributions, the crudeness of those obtained, the arbitrariness of the definitions of kills, the difficulty of computation and the length of time necessary to carry out the computations, one cannot compute the probability of such a kill exactly. Therefore, simplifying physical assumptions were made to obtain approximate results. This is justified since the available vulnerability data for this problem are known to at best from ten to twenty percent and the errors of approximation in the mathematical work will not be as great.

Therefore, in the particular problem discussed in this report it was assumed that:

a) The missile and the airplane move in parallel, horizontal planes and the projection of the direction of the missile on the plane of motion of the airplane forms a line 45° off the nose of the airplane.

b) The distribution of burst points is a trivariate normal distribution in Cartesian coordinates (x,y,z) with a mean (m_1,m_2,m_3) somewhere in the plane of the airplane and 45° off its nose in the direction of the approach of the missile and variances σ_1^2, σ_2^2, and σ_3^2 and zero correlations. (In this problem the origin of the coordinate system is at the center of gravity of the airplane; the x-axis points to the starboard; the y-axis points aft; and the z-axis points aloft.)

c) The velocity of the missile is fixed and the velocity of the airplane is negligible relative to the missile and fragment velocities.

d) There is a region called the "blast" region about the airplane within which if a burst occurs there will result, with probability equal to one, a kill. Outside this region the probability of a kill is assumed to be a function solely of the probabilities of destroying individual vital components.

e) The dynamic distribution of lethal fragments is confined to a conical region about the nose with axis coincident with the missile's longitudinal axis and to a region about the side which is bounded by cones whose axes coincide with the missile's longitudinal axis. Furthermore, it is assumed that fragments are identical in mass, size, shape, and in resultant speed at burst and travel in straight lines after burst. It is also assumed that the dynamic distribution of number of fragments per solid angle subtended from the burst point is uniform in the regions of fragmentation mentioned.

We remark that the assumptions a) and b) are not restrictions imposed by the Lotto method or by any of the methods described in this report; they are assumptions of the particular problem solved. The missile could move in any other straight line path and the probability distribution of the burst points could be different without any serious modification of the mathematical formulation for the ORDVAC or of the coding.

The assumption c) is not as crude as it appears in its statement, for the relative motion of the airplane is actually considered in the problem by a modification of the input vulnerability data.

I.2 The Lotto Method of Solution.

A random sampling procedure for estimating the probability of a kill of an airplane by a warhead under the assumptions a) through e) given at the end of the previous section has been successfully used. It is described in reports by F. G. King [2] and by Stanley Sacks and F. G. King [4] from which some of the introductory material in this report is taken.

Briefly, the procedure is as follows. A table, called a firing record table is made; the headings of its columns include "blast", the names of all the vital components, and "kill". A set of three independent random drawings is made from three different normal populations whose means are m_1, m_2, and m_3, respectively and whose variances are σ_1^2, σ_2^2, σ_3^2, respectively. This triplet determines the burst point. The axes of the distributions do not necessarily coincide with those of the airplane but, in general, could be brought into coincidence by a rotation. Using a scale model of the airplane, it is first determined whether the burst is sufficiently close to the airplane to destroy its structure by blast. If so, a one is tabulated under "blast"; if not, a zero is scored. Then for each vital component successively the probability of destroying it is determined. This probability is zero if the component is outside the lethal fragmentation region of the missile or if the component is shielded from the lethal fragment spray by some part of the airplane, e.g., fuselage, wings, or engines; otherwise, this probability is assumed to be a function solely of the distance of the component from the burst and is given by graphs drawn from the vulnerability data for each component. Then a random number from a uniform population in (0,1) is drawn; if this number exceeds the above probability a zero is scored in the firing record table under the heading of this component; otherwise a one is scored. After this has been done for all components a one or a zero is recorded under the column headed "kill". A one indicates that ones appear either under blast or under a sufficient number of vital components of the same type which if destroyed would result in a kill; a zero indicates the negative of this.

burst point chosen at random

This procedure is then repeated one hundred times using a new random burst point each time. One hundredth of the total number of ones in the kill column is then an estimate, in the sense of the strong law of large numbers of the theory of probability of a kill of the airplane by the warhead. This estimate, being a random variable with a binomial distribution, has a variance given by pq/100 where $p + q = 1$ and p is the probability of a kill. This gives a ratio of standard deviation to mean of $0.1 \sqrt{q/p}$, which is between 5% and 20% when kill probabilities is between 0.8 and 0.2. Thus the percentage random error in this method of computation is comparable to, actually slightly better than, the accuracy of the data.

Random estimates of the percentages of kills due to blast or of kills due to the destruction of a sufficient number of vital components of a given type as well as estimates of the probabilities of destroying any particular component or combinations of components are also obtainable from these firing record tables. Furthermore, if the criteria on numbers of vital components of any type to be destroyed to get a kill are changed, the probability of a kill is obtainable from the same firing table. In addition, using several firing record tables, one is able to estimate kill and other probabilities due to several missiles without constructing new tables. In fact, the estimates of these probabilities could be obtained from one firing table but with higher variances; for k missiles the variances would be slightly less than k times the variances for one missile. This is so because instead of having 100 trials in the average, there would only be 100/k trials in the average. The presence of multiply vulnerable components actually raises the kill probability somewhat and this lowers the expected relative error. ?

II. ORDVAC SOLUTION OF THE PROBLEM

II.3 The Mathematical Model.

In view of the simplifying assumptions a) through e) of I.2, the mathematical quantity estimated by the Lotto method is the integral

$$(3.1) \qquad I = \int_{-\infty}^{\infty} d\Phi_1(x) \int_{-\infty}^{\infty} d\Phi_2(y) \int_{-\infty}^{\infty} f(x,y,z)\, d\Phi_3(z)$$

P(x|X)P(X)

where $f(x,y,z)$ is the probability that a kill has been produced (or a blast kill, or any particular combination of vital components has been destroyed) by a burst at (x,y,z) and $d\Phi_1(x)\, d\Phi_2(y)\, d\Phi_3(z)$ is the probability that a burst occurs at (x,y,z). By the assumption b) of I.2, we have

$$d\Phi_1(x)\, d\Phi_2(y)\, d\Phi_3(z) = \frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3}\, e^{-\frac{1}{2}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2\right]} dx\, dy\, dz.$$

We shall estimate the same integral by three different methods which will be described in II.6. Each of the three methods has in common the same method of estimation of the integrand $f(x,y,z)$ which, in addition to depending on the burst point, (x,y,z), depends on the geometrical representation of the airplane and the vulnerability data.

Criteria for a good geometrical model are that

a) it should represent the airplane, its more bulky components, and the blast region fairly well,

b) it should contain only simple formulas that do not require too many registers in the machine's internal storage and that do not take much computing time to be evaluated,

c) it should lead to simple formulas for the regions shielded by bulky parts of the airplane, and

d) it should lead to simple determinations of whether or not a burst is in the blast region or a vital component is in a shielded region.

Three models have been considered as approximations to the form of the airplane and blast region: A set of ellipsoids to represent the fuselage, wings, engines, and blast region; a set of spheres to represent the above; and a set of cylinders to represent the above except the wings which could be represented by a rectangular parallelipiped. They all satisfy the above four criteria fairly well. The best, however, seems to be the first, the suggestion of J. von Neumann. It seems to satisfy a) better than either of the other two. All three models satisfy b) equally well except that many more spheres than ellipsoids are necessary to represent approximately the volumes in question and, therefore, require more machine storage and computing time. All three involve only quadric surfaces which require only a few products and sums to describe the elemental surfaces and for which it involves only simple discriminations on the sign of a quadratic or a linear form to determine whether or not a point is enclosed by the surface. To determine the regions shielded by bulky parts of the airplane is more difficult using the third model than for the first two. Thus, in view of these considerations, the ellipsoid model was chosen.

Another modification is the replacement of volume-occupying vulnerable components by point components. The spray regions are bounded by cones as in the Lotto method. A vital component in this region is considered to be shielded by a part of the airplane represented by an ellipsoid if it lies outside the ellipsoid and lies within the region inside the part of the cone determined by the burst point and the ellipsoid which is on the other side of the ellipsoid from the burst point. The mathematical criteria for this will be derived in the next section.

II.4 Derivation of Mathematical Criteria for Determining Whether or Not a Vital Component is in the Lethal Fragment Spray.

The probability that a vital component at a point C is destroyed by fragments from a burst at a point P is zero if the component is not in the lethal fragment spray and is a function of the distance $|C - P|$ if the component is in the lethal fragment spray. A vital component at C (See Fig. 1) is assumed to be vulnerable to fragments from a burst at P of a missile whose unit direction vector is V at the time of burst if it is either inside the nose spray cone K_1 or outside both K_1 and the rear cone K_2 and, in addition to either of these conditions, is not shielded or masked from the burst by some part of the airplane represented by ellipsoids, one of which is represented in Fig. 1 by E. If C is inside E it is still considered vulnerable if it is in the spray regions and not shielded by another ellipsoid. (Perhaps a better assumption would be that if C is inside E it is vulnerable only if it is on the same side of the polar plane of P with respect to E as P. However, the way in which the data for the kill probabilities as a function of distance are gathered and averaged militates against this.)

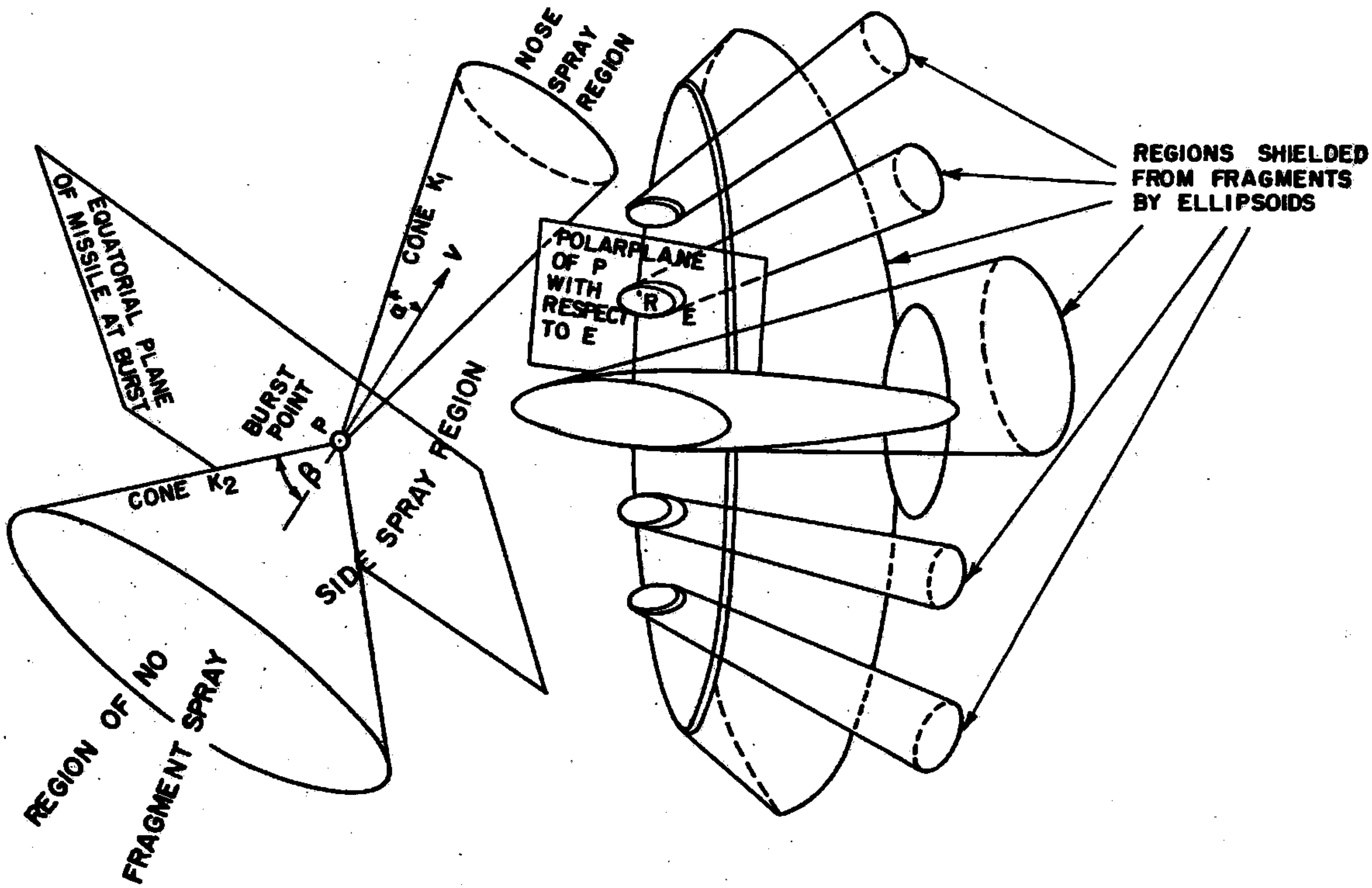

REGIONS SHIELDED FROM FRAGMENTS BY ELLIPSOIDS
NOSE SPRAY REGION
CONE K_1
V
α
POLARPLANE OF P WITH RESPECT TO E
R
E
EQUATORIAL PLANE OF MISSILE AT BURST
BURST POINT
P
β
CONE K_2
SIDE SPRAY REGION
REGION OF NO FRAGMENT SPRAY

FIGURE 1

Fig. 1 represents a dynamic diagram and, therefore, the angle β is greater than the angle α because the momentum of the missile is added to the momentum of the fragments acquired in the burst.

To obtain the mathematical criteria for determining whether a vital component is or is not in the lethal fragment spray we need the equations for the cones K_1, K_2, and K_3, which we shall derive directly from vector considerations. ?

A point $X = (x_1, x_2, x_3)$ is on either nappe of the cone K_1, one of whose nappes is represented in Fig. 1 if $X = R$

$t^2 - R^2 \cos^2\alpha = 0$

$$-K_1(X) = [(X - P) \cdot V]^2 - |X - P|^2 \cos^2 \alpha = 0.$$

It is outside the nappes of the cone if the quadratic form $K_1(X)$ is positive and it is inside if $K_1(X)$ is negative. Similarly, a point X is on either nappe of the cone K_2, is outside the nappes of K_2, or is inside the nappes of K_2 according as the quadratic form

$$-K_2(X) = [(X - P) \cdot V]^2 - |X - P|^2 \cos^2 \beta$$

is zero, is positive, or is negative, respectively.

A point $Y \neq P$ is on the cone K_3, the cone determined by the pencil of lines through the burst point P and tangent to the shielding ellipsoid E, if and only if the line through P and Y is one of these tangent lines. To express this mathematically, let us assume for the while that R the center of the ellipsoid E is also the origin of our coordinate system, and let $X = (x_1, x_2, x_3)$ be a point on the ellipsoid E whose equation in this system is

$$XAX^T = 1 \qquad (4.2)$$

where X^T is the transpose of the row vector X and

$$A = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{1}{b^2} & 0 \\ 0 & 0 & \frac{1}{c^2} \end{pmatrix}$$

in which a, b, and c are the principal semi-axes of the ellipsoid E. If X is a point on E and t is a parameter, then

$$Y = P\,(1-t) + t\,X \tag{4.3}$$

is the equation of a line through P and X. Now Y is on the ellipsoid E if

$$\left[P(1-t) + t\,X\right] A \left[P(1-t) + t\,X\right]^T = 1$$

or, since A is symmetric,

$$t^2(PAP^T - 2\,PAX^T + XAX^T) + 2t(PAX^T - PAP^T) + (PAP^T - 1) = 0. \tag{4.4}$$

Now Y is a tangent point on E if and only if there is a unique t satisfying (4.4), i.e., if

$$(PAX^T - PAP^T)^2 - (PAP^T - 2\,PAX^T + XAX^T)\,(PAP^T - 1) = 0,$$

which when expanded and rearranged yields

$$(PAX^T - 1)^2 - (PAP^T - 1)\,(XAX^T - 1) = 0. \tag{4.5}$$

But since X is on E (4.5) reduces to

$$PAX^T = 1. \tag{4.6}$$

This means that X must lie on the polar plane of P with respect to E. To obtain the equation of the cone K_3 (Obviously, P must be outside E for K_3 to be a real cone.), we let the parameter t in (4.3) range over the real numbers and the variable point X range over the ellipse determined by the ellipsoid E and the polar plane of P with respect to E. To determine the equation of this cone in non-parametric form we eliminate X and t between (4.2), (4.3), and (4.6) for $t \neq 0$, since $t = 0$ yields $Y = P$.

Letting $t \neq 0$ and solving (4.3) for X, we obtain

$$X = P + \frac{1}{t}\,(Y - P) \tag{4.7}$$

which we substitute in (4.2) to get

$$PAP^T + \frac{2}{t}\,PA\,(Y - P)^T + \frac{1}{t^2}\,(Y - P)\,A\,(Y - P)^T = 1. \tag{4.8}$$

Substituting (4.7) in (4.6), we obtain

$$PAP^T + \frac{1}{t} PA (Y - P)^T = 1 \tag{4.9}$$

Subtracting (4.9) from (4.8) and multiplying the result by t, we have

$$PA (Y - P)^T + \frac{1}{t} (Y - P) A (Y - P)^T = 0. \tag{4.10}$$

Eliminating 1/t between (4.9) and (4.10), we arrive at

$$PAP^T (Y - P) A (Y - P)^T - \left[PA (Y - P)^T\right]^2 - (Y - P) A (Y - P)^T = 0.$$

Expanding this and using the symmetry of A, we have

$$\begin{aligned} &PAP^T YAY^T - 2 PAP^T PAY^T + (PAP^T)^2 \\ &- (PAY^T)^2 + 2 PAY^T PAP^T - (PAP^T)^2 \\ &- YAY^T + 2 PAY^T - PAP^T = 0 \end{aligned}$$

or

$$(PAP^T - 1) (YAY^T - 1) - (PAY^T - 1)^2 = 0$$

for the equation of the cone K_3.

Thus a point X is on either nappe of K_3, is outside the nappes of K_3, or is inside the nappes of K_3 according as the quadratic form

$$K_3(X) = (PAP^T - 1) (XAX^T - 1) - (PAX^T - 1)^2 \tag{4.11}$$

is zero, is positive, or is negative respectively. For each shielding ellipsoid there is a different matrix A; therefore, we have a set of quadratic forms of the type of K_3. If there are K shielding ellipsoids, E_k, let us call their respective matrices A_k, $k = 1, 2, \ldots, K$, and the quadratic forms associated with the cone determined by P and the k-th ellipsoid will be designated by $K_{3,k}$. Thus

$$K_{3,k}(X) = (PA_kP^T - 1) (XA_kX^T - 1) - (PA_kX^T - 1)^2. \tag{4.12}$$

It should be recalled that the origin of the coordinate system in the formula is R_k the center of E_k and that the quantities P and X are usually measured relative to the center of gravity of the airplane. Therefore, the appropriate translations should be made before using the formula (4.12). We return now to our original reference coordinate system with its origin at the center of gravity of the airplane.

The fragment spray kill probability for each component is assumed to be different in the nose-spray region (inside K_1) from that in the side-spray region (outside K_1 and K_2) but is assumed to be uniform with respect to the angle subtended from V (See Fig. 1.) in each region. If the point representing a particular component falls in the nose-spray region and is not shielded by any ellipsoid E_k, let us call the procedure of computing the probability of destruction of the component by fragments procedure A; if the component falls in the side spray region and is unshielded, the corresponding procedure will be called procedure B; and if the component falls inside the rear cone, K_2 or is shielded, the corresponding procedure (which is simply recording a zero) will be called procedure C.

The logical steps to determine the choice of the appropriate one of the above procedures are as follows. First the scalar product $(C - P) \circ V$ is formed. If this is positive then the component C lies on the same side of the equatorial plane of the missile at burst as the direction vector of the missile points; if $(C - P) \circ V$ is negative then C is on the other side of the equatorial plane.

If $(C - P) \circ V > 0$, then the component C is either in the nose spray region or side spray region, and it is next determined whether it is shielded by one or more of the ellipsoids E_k, the logic of which determination will be outlined later. If the component is shielded, we pass on to apply procedure C. If it is not shielded, then $K_1(C)$ is formed. If $K_1(C) < 0$, procedure A is applied since C is then in the nose spray and is vulnerable. If $K_1(C) \geq 0$, then C is in the side spray region and procedure B is to be followed.

If $(C - P) \circ V \leq 0$, then C is either in the side spray region or in the rear cone region of no fragment spray. In order to determine in which of these two regions C lies, $K_2(C)$ is formed; if $K_2(C) < 0$, C is in the no spray region and procedure C is applied; if $K_2(C) \geq 0$, it is next determined whether C is shielded or not. If C is shielded, procedure C is then applicable, and if C is unshielded, we go on to apply procedure B.

When C is in a spray region, in order to determine whether it is shielded or not the subsequent logical procedure is followed. Recalling that the center of the k-th shielding elliposid E_k is R_k, $k = 1, 2, \ldots, K$, we form $(P - R_k)\, A_k (C - R_k)^T - 1$. If $(P - R_k)\, A_k (C - R_k)^T - 1 < 0$, then P and C are on opposite sides of the polar plane of P with respect to the ellipsoid E_k and we form

$$K_{3,k}(C) = \left[(P-R_k)\, A_k (P-R_k)^T - 1\right]\left[(C-R_k)\, A_k (C-R_k)^T - 1\right] - \left[(P-R_k)\, A_k (C-R_k)^T - 1\right]^2.$$

If $K_{3,k}(C) < 0$, C is inside the cone $K_{3,k}$ and we form $(C-R_k) A_k (C-R_k)^T - 1$. If $(C-R_k) A_k (C-R_k)^T - 1 > 0$, then C lies outside the ellipsoid E_k; but, as a result of previous discriminations, it is found to be inside $K_{3,k}$ and on the opposite side from P of the polar plane of P with respect to E_k. Hence C is shielded and we pass on to the procedure C.

On the other hand, if $(P-R_k) A_k (C-R_k)^T - 1 \geq 0$, or if $(P-R_k) A_k (C-R_k)^T - 1 < 0$ and $K_{3,k}(C) \geq 0$, or if $(P-R_k) A_k (C-R_k)^T - 1 < 0$, $K_{3,k}(C) < 0$, and $(C-R_k) A_k (C-R_k)^T - 1 \leq 0$, then C is not shielded by E_k because it will be on the same side as P of the polar plane of P with respect to E_k, or it will be on the other side but also outside the cone $K_{3,k}$ or it will be inside $K_{3,k}$ but also inside E_k and thus not shielded according to our assumption. If C is then not shielded by E_k and $k < K$, k is replaced by k + 1 and the procedure is repeated. If k = K then procedure A or B is applied depending on whether C is in the nose spray region or the side spray region.

It should be remarked that the restriction that the matrices A_k in (4.12) be diagonal is particular to the actual problems run on ORDVAC but is not necessary to the derivation of (4.12). If the shielding ellipsoids E_k had axes which were not parallel to the coordinate axes then the A_k would not be diagonal but (4.12) would still be valid. For airplanes with swept back wings such modifications are necessary. It is possible that for delta wings ellipsoidal approximations are too crude. In which case a triangular prism of very small depth could be used with a small increase in memory requirements for the necessary discriminations to determine the shielded regions for each burst point.

The logical flow chart of the application of the criteria of this section for one component is ~~given in Fig.~~ 2.

II.5 Computation of the Kill Probability for a Given Burst Point.

The integrand f(x,y,z) of (3.1), which is the probability that a kill has been produced by a burst at (x,y,z), was programmed for computation by two different methods. One method has the advantage of being very general, but, on the hand, with our specific kill criteria it required about seven times the computing time of the other. The recent addition of a new order, logical "and" to the ORDVAC's list of instructions will by its use make the two computing times nearly the same. Both methods will be described here.

The more general method will be discussed first. If there are M vulnerable components of the airplane, then when a missile explodes, there are 2^M mutually exclusive possible events, besides blast, corresponding to the destruction or non-destruction of each of the components $C_1, C_2, \ldots, C_M$. Each of the possible events may be represented by an M-digit binary integer $\sum_{i=1}^{M} 2^i \beta_i$ where

$$\beta_i = \begin{cases} 1 & \text{if } C_i \text{ is destroyed} \\ 0 & \text{if } C_i \text{ is not destroyed.} \end{cases}$$

Let us define a partial ordering relation, $(\leq)$, among the M digit binary integers such that

$$\sum_{i=1}^{M} 2^i \beta_i \ (\leq) \ \sum_{i=1}^{M} 2^i \gamma_i.$$

if and only if $\beta_i \leq \gamma_i$ for each $i = 1, 2, \ldots, M$. Let S be the subset of the 2^M possible M-digit binary integers which represent kills. Let S^* be that subset of S such that a) if s is in S then there exists an element s^* in S^* such that $s^*(\leq)$ s and b) if s^* is in S^* then there does not exist another element, s, of S such that $s(\leq) s^*$. If r is an M-digit binary integer, let us define a function $\epsilon(r)$ such that

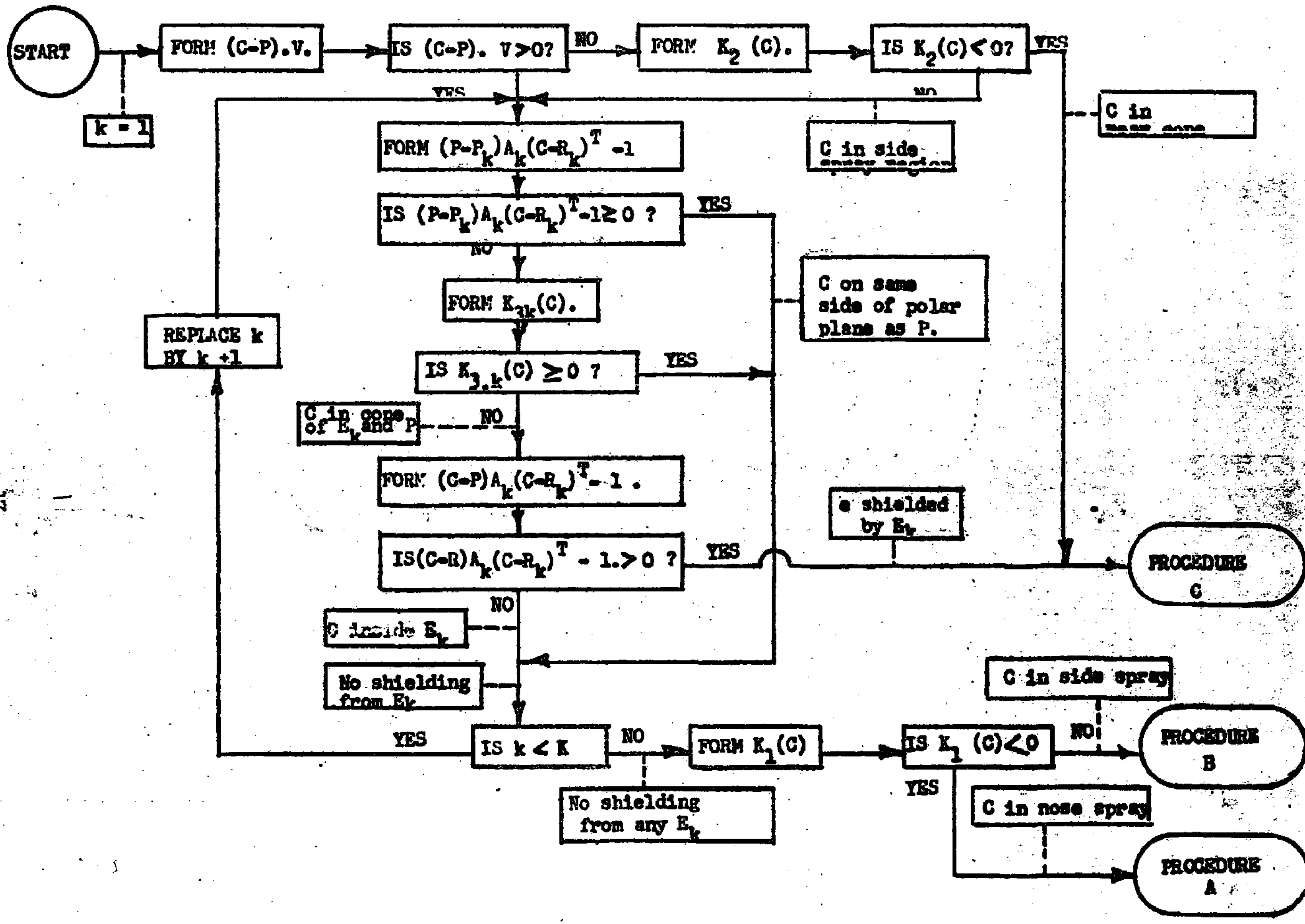
$$\epsilon(r) = \begin{cases} 1 & \text{if there exists an } s^* \text{ in S such that } s^*(\leq)r \\ 0 & \text{if there does not exist an } s^* \text{ in S such that } s^*(\leq)r. \end{cases}$$

We remark that for $\epsilon(r) = 1$ it is necessary and sufficient that r be in S.

The probability p_i, of destruction of the component C_i is, in the problem handled, a function of the distance, D, from the component C_i to the given burst point (x,y,z) the density of the lethal fragment spray, and the vulnerable area of the component, A_v. It is given by

$$p_i = 1 - e^{-a A_v D^{-2}}$$

where a is a constant proportional to the fragment density. The vulnerable area A_v is obtained from experimental data and for ORDVAC computations was fitted by a curve composed of segments of parabolas and straight lines. This fitting was done by W. Barkley Fritz.



CORRECTED VERSION OF FIGURE 2 OF BRL REPORT NO. 867

If p_i is the probability of destruction of the component C_i by a burst at the given point (x,y,z) and if $r = \sum_{i=1}^{M} 2^i \beta_i(r)$, then the probability, $P(r)$, of occurrence of the event represented by the binary integer, r, is given by

$$P(r) = \prod_{i=1}^{M} \left\{ \beta_i(r)\, p_i + \left[1 - \beta_i(r)\right] (1 - p_i) \right\}$$

and the probability, P_F, of a kill due to fragments from a burst at (x,y,z) is given by

$$P_F = \sum_{r=0}^{2^M-1} \epsilon(r)\, P(r).$$

This method is very general, but, for the specific kill criteria in the actual problem solved on ORDVAC, a second method was also used in which it was possible to decrease the large number of operations required by avoiding the comparisons involved in the relation ($\leq$) and reducing the total number of probabilities needed to compute P_F. The efficiency of the second method depends on the fact that in the cases treated the set of vital components was composed of a number of disjoint subsets containing only components of the same type and the number of any one type was small; furthermore, for each type there exists a fixed number of vital components which must be destroyed in order to achieve a kill. If there are J different types and if Q_j is the probability of killing at least the required number of vital components of the type, J_j, sufficient to produce a kill, then

$$P_F = 1 - \prod_{j=1}^{T} (1 - Q_j).$$

Given the probabilities, p_i, since the number of components of each type is small, the computation Q_j is very simple. For example, if J_1 contained only the components C_1 and C_2, and if the destruction of only one were sufficient for a kill then $Q_1 = p_1 + p_2 - p_1p_2$. If J_1 contained only the components C_1, C_2, and C_3, and if the destruction of only two were sufficient for a kill then
$Q_1 = p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3$.

Finally, if P_B is the probability that the airplane's structure is destroyed by blast from a burst at the given point (x,y,z), then the integrand of (3.1) is given by

$$f(x,y,z) = P_B + (1-P_B)\,P_F = \begin{cases} 1 & \text{if } (x,y,z) \text{ is in the blast region} \\ P_F & \text{if } (x,y,z) \text{ is not in the blast region.} \end{cases}$$

$$= 1-(1-P_B)(1-P_F)$$

II.6 Choices of Methods of Evaluation of the Kill Probability Integral.

Several methods of quadrature to evaluate the kill probability integrals given by (3.1) were considered. These methods fall into two general classes, random sampling or Monte Carlo procedures and systematic numerical integration.

Criteria for choosing any method over others are usually based on the accuracy desired in the value of the integral (3.1), on the accuracy and speed available in the machine, and machine internal storage requirements for each method. In this problem since sufficiently accurate and sufficiently fast methods with about the same machine internal storage requirements are available in both the above two general classes, storage requirements are of minor importance in the making of choices among the methods considered. The programs actually used occupied nearly all of the ORDVAC storage, but, if it were necessary, the same procedures could be coded to take a somewhat smaller storage - about 850 or so words.

In random sampling procedures since a correct statement of the answer is that the answer is, say x with probability p, the criterion assigning greater accuracy to a particular random sampling method than to another usually that the first method have a smaller variance than the second. In choosing one sampling method over another we shall use this criterion in addition to the criterion of speed.

Among the random sampling integrating procedures three were given some consideration for possible use on ORDVAC; let them be called RSIP-I, RSIP-II, and RSIP-III.

RSIP-I is the Lotto procedure described in I.2 with the mathematical model given in II.3.

In RSIP-II a sequence of N points $\{(x_n, y_n, z_n)\}$, $n = 1, 2, \ldots, N$, is chosen from a trivariate normal distribution with mean (m_1, m_2, m_3) and variance $(\sigma_1, \sigma_2, \sigma_3)$. Then an estimate, in the sense of the strong law of large numbers, of the integral (3.1) is given by

$$\frac{1}{N}\sum_{n=1}^{N} f(x_n, y_n, z_n).$$

In RSIP-III a sequence of N points $\{(x_n, y_n, z_n)\}$, $n = 1, 2, \ldots, N$, is chosen from a trivariate uniform distribution in the region defined by

$$|x-m_1| \le 4\sigma_1, \ |y-m_2| \le 4\sigma_2, \ |z-m_3| \le 4\sigma_3.$$

Then an estimate, in the sense of the strong law of large numbers, of the integral (3.1) is given by

$$\frac{1}{N}\left(\frac{32}{\pi}\right)^{\frac{3}{2}} \sum_{n=1}^{N} f(x_n,y_n,z_n)\, e^{-\frac{1}{2}\left[\left(\frac{x_n-m_1}{\sigma_1}\right)^2 + \left(\frac{y_n-m_2}{\sigma_2}\right)^2 + \left(\frac{z_n-m_3}{\sigma_3}\right)^2\right]}.$$

The ranges of $8\sigma_1$, $8\sigma_2$, and $8\sigma_3$ for the random variables x_i, y_j, and z_k were chosen because the probability mass within 4σ of the mean in the univariate normal distribution is 0.9999365. Since accuracies of several percent in the answers are more than are necessary in this problem, ranges of seven σ's or even six σ's could be used instead of eight σ's. In the normal distribution the probability mass within 3.5σ of the mean is 0.999535 and it is 0.997300 within 3σ of the mean.

The random number generation used in these schemes is discussed in the Appendix.

Among the systematic numerical integrating procedures two were considered; let them be called SNIP-I and SNIP-II. Because the accuracy desired in the problem is of the order of several percent, simple procedures are sufficient. Therefore, each of these two procedures considered were three-dimensional Riemann sums.

In SNIP-I the summands are evaluated at L^3 points at the geometric centers of elemental cubes of equal volume in the physical space. For the same reasons expressed in the description of RSIP-III the region of integration is restricted to

$$|x-m_1| \le r\sigma_1, \ |y-m_2| \le r\sigma_2, \ |z-m_3| \le r\sigma_3,$$

where r may be chosen as 3, 3.5, or 4. The integral (3.1) is estimated by

$$\left(\frac{r}{L}\sqrt{\frac{2}{r}}\right)^3 \sum_{i=1}^{L} \sum_{j=1}^{L} \sum_{k=1}^{L} f(x_i,y_j,z_k)\, e^{-\frac{1}{2}\left[\left(\frac{x_i-m_1}{\sigma_1}\right)^2 + \left(\frac{y_j-m_2}{\sigma_2}\right)^2 + \left(\frac{z_k-m_3}{\sigma_3}\right)^2\right]}$$

where

$$x_i = m_1 - r\sigma_1\left(1 - \frac{2i-1}{L}\right)$$

$$y_j = m_2 - r\sigma_2\left(1 - \frac{2j-1}{L}\right)$$

and

$$z_k = m_3 - r\sigma_3\left(1 - \frac{2k-1}{L}\right).$$

In SNIP-II the summands are evaluated at L^3 points at the probability mass centers of elemental cubes of equal probability mass in a three-dimensional probability space whose probability mass distribution is a trivariate normal distribution. The integral (3.1) is estimated by

$$\frac{1}{L^3} \sum_{i=1}^{L} \sum_{j=1}^{L} \sum_{k=1}^{L} f(m_1 + \sigma_1 t_i,\ m_2 + \sigma_2 t_j,\ m_3 + \sigma_3 t_k)$$

where

$$t_\nu = t_{L-\nu}$$

and for $\nu \leq L/2$

$$\sqrt{\frac{1}{2}} \int_0^{-t_\nu} e^{-\frac{t^2}{2}}\, dt = \frac{L + 1 - 2\nu}{2L}.$$

II.7 Comparison of Methods of Evaluation of the Kill Probability Integral.

In this section the methods listed in II.6 will be compared on the basis of theoretical accuracy, speed and memory requirements.

The random sampling integrating procedures will be considered first.

If $\{X_i\}$ is a sequence of independent identically distributed random variables and $G(x)$ is a function of a very general class (Baire functions are included in this class.), then

$$\mathrm{Var}\left\{\frac{1}{n}\sum_{i=1}^{n} G(X_i)\right\} = \frac{1}{n}\mathrm{Var}\left\{G(X_i)\right\} = \frac{1}{n}E\left\{G^2(X_i)\right\} - \frac{1}{n}E^2\left\{G(X_i)\right\}.$$

This formula will be used to determine the respective theoretical variances of the three random sampling integrating procedures. The respective variances will be designated as Var_I, Var_{II}, and Var_{III}.

In RSIP-I, $G(X_i)$ is a Bernoulli random variable which takes on the value 0 with probability I and the value 1 with probability 1-I, where I is the kill probability integral (3.1). Thus

$$\mathrm{Var}_I = \frac{1}{N}(I - I^2). \tag{7.13}$$

In RSIP-II, X_i is a three-dimensional random variable with a trivariate normal distribution with mean equal to (m_1, m_2, m_3) and variances equal to $(\sigma_1, \sigma_2, \sigma_3)$ and $G(X_i) = f(x_i, y_i, z_i)$, the integrand in (3.1). Therefore,

$$(7.14) \quad \mathrm{Var}_{II} = \frac{1}{N}\left[\int_{-\infty}^{\infty} d\,\Phi_1(x) \int_{-\infty}^{\infty} d\,\Phi_2(y) \int_{-\infty}^{\infty} f^2(x,y,z)\, d\,\Phi_3(z) - I^2\right]$$

In RSIP-III, X_i is a three dimensional random variable with a trivariate uniform distribution in

$$|x-m_1| \leq r\sigma_1, \quad |y-m_2| \leq r\sigma_2, \quad |z-m_3| \leq r\sigma_3$$

where the value of r could conveniently be 3, 3.5, or 4.

Then

$$G(X_i) = \left(\frac{2r^2}{\pi}\right)^{3/2} f(x_i,y_i,z_i)\, e^{-\frac{1}{2}\left[\left(\frac{x_i-m_1}{\sigma_1}\right)^2 + \left(\frac{y_i-m_2}{\sigma_2}\right)^2 + \left(\frac{z_i-m_3}{\sigma_3}\right)^2\right]}$$

and

$$(7.15) \quad \mathrm{Var}_{III} = \frac{1}{N}\left\{\int_{-r\sigma_1}^{r\sigma_1} d\,\Phi_1(x) \int_{-r\sigma_2}^{r\sigma_2} d\,\Phi_2(x) \int_{-r\sigma_3}^{r\sigma_3} \left(\frac{2r^2}{\pi}\right)^{3/2} e^{-\frac{1}{2}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_1}{\sigma_3}\right)^2\right]} f^2(x,y,z)\, d\,\Phi_3(y) - I^2\right\}$$

The only difference in the expressions for Var_I and Var_{II} is in the power of the integrand, $f(x,y,z)$, in the first members of the right hand sides of (7.13) and (7.14). In the former $f(x,y,z)$ appears to the first power, while in the latter it is squared. Hence, since $f(x,y,z)$ is the probability of a kill or some other desired event from a burst at the point (x,y,z) and, therefore, is between zero and one, $\mathrm{Var}_{II} \leq \mathrm{Var}_I$.

In the Lotto method it has been customary to choose $M = 100$. For this the root mean square relative error is $0.1\sqrt{(1-I)/I}$, implying that if $\alpha\%$ is a bound on the allowable root mean square relative error then

$$(7.16) \qquad (1 + \alpha^2/100)^{-1} \leq I.$$

This implies, for example, that if the root mean square relative errors are not to exceed 10% then I must not be less than 1/2; if the root mean square errors are not to exceed 20% then I must exceed 1/5. (Of course, choosing M to be larger will reduce the error, so that if small values of I are to be computed with accuracy more extensive sampling is needed.) To test the random numbers used as well as to compare RSIP-I and RSIP-II fifty runs of the problem were made using RSIP-II with different and independently randomly chosen values of r_0 at the request of F. G. King. The values of the variances obtained for almost all of the probabilities of kills of component combinations indicated that approximately 1.5 to 3.2 times as many burst points would have been required to produce the same accuracy if RSIP-I had been used instead of RSIP-II. For the probabilities of kills due to destruction of a sufficient number of one particular type of vital component from 5 to 8 times the number of burst pointe used in RSIP-II would have been needed to get the same accuracy if one used RSIP-I. However, since these probabilities are small and contribute only a little to the total probability of a kill, great accuracy in these small probabilities is certainly not necessary unless one were strongly interested in the particular components' probability of destruction.

The difference $\mathrm{Var}_I - \mathrm{Var}_{II}$ is given from formulas (7.13) and (7.14) as

$$\mathrm{Var}_I - \mathrm{Var}_{II} = \frac{1}{N} \int_{-\infty}^{\infty} d\Phi_1(x) \int_{-\infty}^{\infty} d\Phi_2(y) \int_{-\infty}^{\infty} f(1-f)\, d\Phi_2(z).$$

Therefore, from this formula one sees that if most of the probability mass is near where f(1-f) is near its maximum, i.e., where $f = 1/2$, then the difference $\mathrm{Var}_I - \mathrm{Var}_{II}$ is of the order of Var_I. In the case of singly vulnerable components or multiply-vulnerable components physically close together the bulk of the total kill probability on a component combination is obtained from points near the components, or near the component combination as the case may be, where f is near 1.

The storage requirements and the computing time required for RSIP-I is about the same for RSIP-II for the same number of bursts because of the extra randomization in RSIP-I while their printing times are identical provided that the sequences of zeros and ones which are the outcomes of each burst in RSIP-I are not printed. On the other hand, if these zeros and ones are to be recorded as in the hand Lotto method the printing time in RSIP-I is greater than in RSIP-II. Furthermore, if the same root mean square relative error is desired in RSIP-I as in RSIP-II, as has been pointed out, for moderate values of (3.1) at least twice the computing time of RSIP-II is required. Therefore, on the basis of these considerations RSIP-II is recommended by the present authors over RSIP-I, provided that firing record tables are not required to be printed for use in other problems.

The storage requirements for RSIP-II are slightly higher than for RSIP-III. The computing time required for RSIP-III is approximately 2/3 that of RSIP-II for the same number of bursts. This is caused by the fact that RSIP-II requires random numbers chosen from normal distributions while RSIP-III requires random numbers chosen from a uniform distribution and the method used on ORDVAC to produce normally distributed random numbers transformed uniformly distributed random numbers into normally distributed random numbers and used a routine to compute logarithms as part of the transformation; this routine requires a good fraction of the computing time.

However, Var_{II} is somewhat less than Var_{III}. The only difference between the first members of the right hand side of (7.14) and of (7.15) is that in (7.15) there appears the extra factor

$$(7.17) \qquad \left(\frac{2r^2}{\pi}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2\right]}.$$

For $r = 3$ and

$$(7.18) \qquad \left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2 < 5.24$$

this factor exceeds unity and is less than unity elsewhere. Furthermore, $f^2(x,y,z)$ is generally larger in the region is determined by (7.18) than elsewhere where it is probably very small because it is the square of the kill probability for a burst at (x,y,z) and generally decreases away from the origin which is not far from the mean burst point, certainly very much closer than $5.24 \min(\sigma_1,\sigma_2,\sigma_3)$. Finally, since

$$\frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \iiint\limits_{\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2 > 5.24} e^{-\frac{1}{2}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2\right]} dx\,dy\,dz$$

$$= \frac{1}{(2\pi)^{3/2}} \iiint\limits_{x^2+y^2+z^2 > 5.24} e^{-\frac{1}{2}(x^2+y^2+z^2)} dx\,dy\,dz = \sqrt{\frac{2}{\pi}} \int_{\sqrt{5.24}}^{\infty} \rho^2 e^{-\frac{1}{2}\rho^2} d\rho$$

$$= \sqrt{\frac{2}{\pi}} \left[2.289\, e^{-2.62} + \int_{5.24}^{\infty} e^{-\frac{1}{2}\rho^2} d\rho\right] = 0.155$$

and because of the foregoing remarks about $f^2(x,y,z)$, a large fraction of the value of the first members of the right hand sides of (7.14) and

(7.15) is contributed by the integration over the region determined by (7.18) in the case $r = 3$. For $r = 4$ the factor (7.17) exceeds unity in the region determined by

$$(7.19) \qquad \left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2 < 6.96$$

and is less than unity elsewhere. In view of the remarks made about $f^2(x,y,z)$ and since

$$\frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \iiint\limits_{\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2 > 6.96} e^{-\frac{1}{2}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 + \left(\frac{y-m_2}{\sigma_2}\right)^2 + \left(\frac{z-m_3}{\sigma_3}\right)^2\right]} dx\, dy\, dz = 0.073,$$

by far most of the contribution to the first member of the right hand sides of (7.14) and (7.15) comes from the integration over the region determined by (7.19). The situation is similar for $r = 3.5$ or any other value of r between 3 and 4. Moreover, in the neighborhood of the mean the contribution to the first member of the right hand side of (7.15) is from 18 to 32 times the contribution to the first member of the right hand side of (7.14) for values of r between 3 and 4. Consequently, in all practical cases $Var_{II} \leq Var_{III}$.

Since the computing times and memory requirements are about the same order of magnitude for the three random sampling integrating procedures the smallest variance for the same number of sampling points is the deciding criterion, especially in view of the fact that, in order to achieve the same accuracy with the methods with higher variances for the same number of sampling points, more points, and hence more computing time, are needed. Therefore, RSIP-II is recommended and was actually run on ORDVAC.

The two systematic numerical integrating procedures will be considered next.

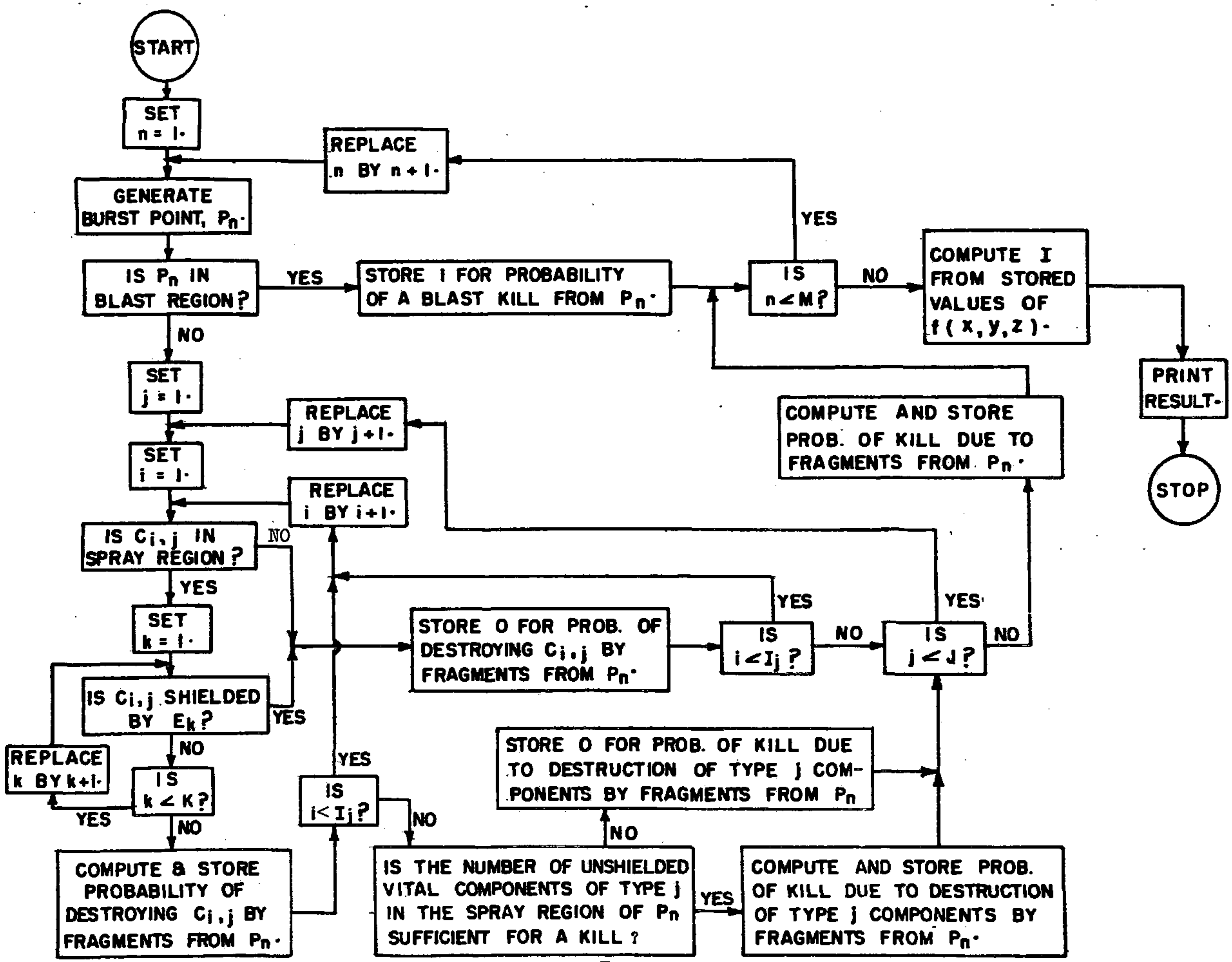
In SNIP-I, using volumes 6σ's, 7σ's, and 8σ's on an edge and choosing 4^3, 5^3, and 6^3 points at the centers of equal volume elements whose sides are parallel to those of the large volume, the kill probabilities were found to be in fair agreement; the variations were approximately the same as the root mean square relative errors in the random sampling procedures. However, the agreement among the blast kills was poor; this was apparently due to the fact that the points chosen lie in planes parallel to the plane of the wings of the airplane and are separated by quite a few feet thus giving a poor sample of points in the blast regions. Tried out in a few cases with analytically integrable integrands as well, SNIP-I gave poorer results than SNIP-II. Therefore, SNIP-I is not recommended over either the random sampling procedures or SNIP-II, even though its computing time is about two-thirds that of the random sampling procedures for the same number of points. SNIP-I is also slightly longer than SNIP-II in computing time.

In SNIP-II 4^3, 5^3, and 6^3 points were chosen spaced as indicated in the previous section; these were run for all five sets of $\sigma = (\sigma_1, \sigma_2, \sigma_3)$. For the trials using 4^3 points a majority of the results were within the observed standard deviations of the means in RSIP-II and no results were in bad agreement with those of RSIP-II. The results of the trials using 5^3 and 6^3 points were very well within the observed standard deviations of the means in RSIP-II in almost all cases. Furthermore, the results using 5^3 and 6^3 points agreed very closely with each other, in several extremely good cases agreeing to within three or four units in the third significant figure. In general, the differences between the results using 5^3 and 6^3 points were about half, or less, the observed standard deviations of the means in RSIP-II using averages of 200 trials. These results are taken to indicate that the probabilities obtained by SNIP-II using 125 burst points are better than those obtained by RSIP-II using 200 points. Furthermore, the computing time for SNIP-II is about 1.4 minutes for 64 points for each set of σ's, about 2.2 minutes for 125 points for each set of σ's, and about 5 minutes for 216 points for each set of σ's, while in RSIP-II about 4.0 minutes are required for 100 points for each set of σ's. The values quoted were actually timed; for the smaller σ's the blast effects were higher and less time was needed because the loops involving the determination of kills by destruction of components were omitted for many points but the times were higher whenever the blast effects were low for the converse reason. The results of these investigations indicates that for one-missile encounters SNIP-II is recommended over SNIP-I and the random sampling procedures.

II.8 Basic Logical Flow Chart.

The procedures discussed in the previous sections are parts of the basic program whose logical flow chart is approximately that given in Fig. 3 and which we now describe briefly.

Let n be the index of a burst point and M be the total number (usually 100 for the Lotto method) to be used in the computation of I. Let j be the index of the type of vital component and J be the total number of types. Let i be the index of a component of which there will be I_j of type j. Thus, $C_{i,j}$ is the i-th component of the j-th type, $i \leq I_j$, $j \leq J$.



START
SET n = 1.
REPLACE n BY n + 1.
GENERATE BURST POINT, Pn.
IS Pn IN BLAST REGION?
YES
STORE 1 FOR PROBABILITY OF A BLAST KILL FROM Pn.
IS n < M?
YES
NO
COMPUTE I FROM STORED VALUES OF f(x, y, z).
PRINT RESULT.
STOP
NO
SET j = 1.
REPLACE j BY j + 1.
SET i = 1.
REPLACE i BY i + 1.
COMPUTE AND STORE PROB. OF KILL DUE TO FRAGMENTS FROM Pn.
IS Ci,j IN SPRAY REGION?
NO
YES
SET k = 1.
IS Ci,j SHIELDED BY Ek?
YES
NO
REPLACE k BY k+1.
IS k < K?
YES
NO
COMPUTE & STORE PROBABILITY OF DESTROYING Ci,j BY FRAGMENTS FROM Pn.
STORE 0 FOR PROB. OF DESTROYING Ci,j BY FRAGMENTS FROM Pn.
IS i < Ij?
YES
NO
IS j < J?
YES
NO
IS i < Ij?
YES
NO
STORE 0 FOR PROB. OF KILL DUE TO DESTRUCTION OF TYPE j COMPONENTS BY FRAGMENTS FROM Pn
NO
IS THE NUMBER OF UNSHIELDED VITAL COMPONENTS OF TYPE j IN THE SPRAY REGION OF Pn SUFFICIENT FOR A KILL?
YES
COMPUTE AND STORE PROB. OF KILL DUE TO DESTRUCTION OF TYPE j COMPONENTS BY FRAGMENTS FROM Pn.

FIGURE 3

Initially n is set equal to 1. Then a burst point P_n is produced as a triplet of numbers (x_n, y_n, z_n) generated by either "random" sampling or by systematic methods as described in II.6, II.7, and the Appendix. Then it is determined whether or not the point is inside at least one of the ellipsoids, the sum of which determines the blast region. If the point is within the blast region, then in the register reserved for the integrands, $f(x,y,z)$, a 1 is stored for the probability of kill from a burst at P_n; otherwise, the probability of kill is determined from the probability of kill due to fragments as follows.

We begin with determining the probability of destroying the first component of the first type by fragments from a burst at P_n; thus, initially, the index of the type, j, is set equal to 1 and i is also set equal to 1. Then, using the formulas of II.4, it is determined whether $C_{i,j}$ is in the conical spray regions (See Fig. 1.). If it is, then it is determined whether or not $C_{i,j}$ is shielded by any of the K ellipsoids, E_k. If $C_{i,j}$ is not shielded by any of the K ellipsoids then, using vulnerability data and the distance between $C_{i,j}$ and P_n, one computes and stores the probability of destroying $C_{i,j}$ by fragments from P_n. Now, if $i < I_j$, i.e., not all components of type j have been examined, i is replaced by i + 1 and the loop (or an alternate one to be described below) is repeated for $C_{i+1,j}$. If, on the other hand, $i = I_j$ then one determines whether or not the number of unshielded vital components of type j is sufficient for a fragment kill from P_n. If it is, then one computes from combinatorial formulas and stores the probability of a kill due to destruction of type j components by fragments from P_n. If it is not, then this probability is zero and 0 is stored.

If, on the other hand, $C_{i,j}$ is not in the spray region or is shielded by one of the K ellipsoids then a 0 is stored for the probability of destroying $C_{i,j}$ with fragments from P_n. Now, if $i < I_j$, the component $C_{i+1,j}$ replaces $C_{i,j}$ for a tour through either this loop or the above described loop and, if $i = I_j$ and $j < J$ the procedure is repeated with j + 1 replacing j until j = J. Then, using the stored probabilities of obtaining kills by destroying sufficient numbers of each of the different types of components in standard combinatorial formulas, the probability of a kill due to fragments from P_n is computed and stored in the register reserved for $f(x,y,z)$.

Thus, at this stage we have in the register for $f(x, y, z)$ the probability of a kill from a burst at P_n, which is either 1 if it is a blast kill or less if it is a fragment kill. This procedure is repeated until n = M, each time accumulating $f(x_n, y_n, z_n)$ in the same register. Then, multiplying this sum by the proper constants (Since I is a probability integral and since different methods of integration may be used some normalization is necessary.), we obtain I, which is then printed and the program halts.

Many things have not been included in this flow diagram for the sake of brevity and because the special demands of each program introduce details that vary from problem to problem. Segments of this flow chart have been amplified in Fig. 2. Things such as determining and printing separately the probabilities of kill of various components have been omitted since it is not difficult to see how they could be incorporated in this program (and actually have been in several problems solved.)

III. CONCLUDING REMARKS

III.9 General Remarks on the Methods and Generalization.

It takes about ten days using the Lotto method by hand to produce the firing record tables and final probabilities for five different sets of variances in the trivariate normal distribution for the burst point. Machine time on ORDVAC, which included printing time and time of input from IBM cards, to produce with greater accuracy than in the Lotto method the same data exclusive of the firing record tables varied from 3.2 to 3.8 minutes using SNIP-II with 125 points and from about 5 to 5.5 minutes using RSIP-II with 100 points. The variations within each method were due to the variations in the number of times the program had to investigate fragment kills (i.e., when the burst point was not in the blast region), the fragment kill loop in the program (See Fig. 3.) being much more complicated than the blast kill loop. RSIP-I (the Lotto method on the machine) would take slightly longer. However, this does not include coding and code-checking time, which took a few weeks. Nevertheless, since it is foreseen that the problem is of a recurring type, this time is considered as initial overhead as the original formulation of the Lotto method and construction of models must also have been. For subsequent re-runs of the problem with different initial data it is not necessary to repeat the coding; only a small amount of time is required.

An additional advantage in accuracy in the ORDVAC solution over the Lotto method appears in the consistency of the decision as to when the burst is in the blast region. In the Lotto method the operators determine by eye whether or not a burst point is in the blast region, the doubtful cases to be resolved by the use of mathematical formulas; but for psychological reasons these seldom occur. (Mr. King has suggested that the placement on the model of a wire mesh form outlining the blast region would eliminate this indeterminacy in the hand method.)

On the other hand, in the formulation of the problem for ORDVAC the boundary surface of the blast region is represented by a mathematical surface of second degree such that simple discriminations on the sign of a quadratic form definitely determine whether or not the burst is inside the blast region.

The firing record tables in the Lotto method have an advantage in that a change in kill criteria does not mean that new tables must be made. With the already made tables within several hours new probabilities can be computed. Furthermore, the acquisition of large numbers of firing record tables represent large amounts of data which can be, and have actually been, used to get the probabilities of kills in engagements involving several missiles. (Of course, this assumed that the cumulative damage due to bursts from different missiles on a component is not enough to destroy the component unless the damage inflicted by at least one of the missiles on the component is sufficient to destroy it. This would also be an assumption of most practical formulations.) The former advantage can be offset somewhat in the ORDVAC because of the short running times and of the few hours necessary to produce a tape or cards modifying the input kill criteria. On the other hand, no formulation for several missiles has been attempted for ORDVAC and, therefore, no practical comparison between the methods of RSIP-I and RSIP-II are now possible; but it seems likely that for a large number of missiles RSIP-I would be preferred over RSIP-II.

Further generalizations of some sort in the problem seem to be possible in the formulation for the ORDVAC solution. A somewhat more general angular distribution of fragments is possible as well as are a distribution of directions of approach of the missile and some consideration of the directional effect of blast from an exploding moving missile. Clearly, the constants in the formulas for the probabilities of destruction of components by fragments are also possible to be changed with little delay in coding or computing.

It should be remarked that recently there has been added the logical "and" to the list of automatic ORDVAC orders. This permits the rapid use of the more general method described first in II.5, based on the partial ordering ($\leq$), for determining whether a certain combination of components destroyed constitutes a kill or not. The method as originally coded not using this order required a very large amoung of time simply for shifting in order to make the digit-by-digit comparisons necessary in determining whether or not the relation ($\leq$) is satisfied. The elimination of this shifting makes the time required about the same as that required for the less general method described in II.5.

The authors wish to acknowledge gratitude to Mr. F. G. King for many preliminary discussions acquainting them with the problem, to Dr. Saul Gorn for helpful suggestions, in particular, in the geometry of the problem, and to Mr. Frank Lerch for machine information which helped the authors crystallize the mathematical formulation of the problem.

10. The Generation of the Random Numbers.

The so-called "random" numbers used in the random sampling procedures are not random in the strict mathematical sense of the word, but rather they possess to some degree only some of the properties of truly random numbers. Density, frequency of occurence of certain digit combinations in certain positions, and contingencies were some of the properties investigated and compared with the theoretical behavior of truly random numbers by means of χ^2 tests. More properly these random numbers which are generated in a completely deterministic manner are sometimes called pseudo-random numbers, but for economy of expression in this report they will still be called random numbers.

The following method of generating the random numbers directly in our problems is similar to the procedure described by D. H. Lehmer (p. 144 [3]) and to the procedure devised by Olga Taussky-Todd for SEAC. It produces sequences of numbers approximating uniformly distributed random numbers in (0,1) and has the advantages of requiring very few orders and of producing sequences having a very long period (2^{37}) and satisfying very well certain so-called random number tests.

Let ρ_o be an arbitrary odd number satisfying $1 \leq \rho_o \leq 2^{39} - 1$. Define

$$\rho_{n+1} \equiv 5^{13} \rho_n \pmod{2^{39}}, \ n = 0, 1, 2, \ldots,$$

and such that $1 \leq \rho_n \leq 2^{39} - 1$ for every n. Then define

$$r_n = 2^{-39} \rho_n, \ n = 0, 1, 2, \ldots .$$

The sequence $\{ r_n \}$ is the desired random number sequence whose distribution is very close to the uniform distribution in (0,1). On ORDVAC this is simply achieved by multiplying r_n by 5^{13} using double precision, i.e., using two registers (78 binary places) for the product, and in such a manner that the integral part of 5^{13} r_n falls in one register and the fractional part, which is r_{n+1}, falls in the other and is ready to be used in the problem as well as to generate r_{n+2}.

The modulus 2^{39} was chosen because the ORDVAC register has 39 binary digits and the obtaining of the remainder of the division of 5^{13} by 2^{39} is achieved immediately by omitting the first register in the double precision division. The multiplier 5^{13} was chosen because it does not exceed 2^{39} and thus 5^{13} ρ_n fits entirely in the two registers reserved for it; on the other hand, 5^{13} has a reasonable selection of digits and is large enough for ρ_{n+1} and ρ_n not to appear correlated. Furthermore, in order to insure the longest possible period with this sort of scheme the multiplier should be congruent to 1 with respect to the modulus 4, which requirement is satisfied by 5^{13}.

It is perhaps worthwhile to remark that this process is easily generalized for any high-speed digital computing machine. For a machine whose registers contain n digits to the base β, the modulus should be β^n instead of 2^{39}; the multiplier used in place of 5^{13} should be large but less than β^n and if $\beta = 2$ the multiplier should be congruent to 1 with respect to the modulus 4 in order to insure the longest possible period (β^{n-2}). If $\beta \neq 2$, a special analysis for each machine is necessary to determine to which residue classes the multiplier should belong.

Using elementary number-theoretic methods, one can show that this procedure on ORDVAC will produce a succession of exactly 2^{37} different odd numbers, ρ_n and will then continue to repeat the sequence over again. This implies that exactly half of all the odd numbers in the interval $1 \leq \rho_n \leq 2^{39} - 1$ will appear in each period; the particular set of odd numbers appearing will depend on the particular choice of ρ_o. Furthermore, one can easily show that, if, for some n, $\rho_n = \rho$, then there exists no k such that $\rho_{n+k} = \rho + 2$ in the same sequence: thus, the two sets of odd numbers which may be produced interlace.

Using the iterative procedure $\rho_{n+1} \equiv 5^{17} \rho_n \pmod{2^{42}}$, $\rho_o = 1$, to produce pseudo-random numbers on the SEAC, the National Bureau of Standards made some fairly extensive and exhaustive tests whose results indicated very good agreement with what could be expected from truly random numbers. Since our method is very similar and their results were so good, not so extensive tests were performed on the sequences produced on ORDVAC.

With $r_o = 1 - 2^{-39}$ and $r_o = 0.5478126193$ two sequences of 4096 numbers were produced. The number of zeros in each of the following places of the binary representations of the numbers of the sequences: 2nd, 3rd, 5th, 7th, 11th, 18th, and 24th, was counted for each sequence. Values of $\chi_o^2 = 2.87$ and $\chi_{o_2}^2 = 7.48$ respectively were obtained. These values are exceeded by a χ^2 variable with seven degrees of freedom with probabilities of 0.82 and 0.39 respectively. The number of occurrences of 10 in the 4th and 5th and in the 15th and 16th places of the binary representations of the numbers in each sequence was counted, giving

1067 and 1057 for the sequence beginning with $r_o = 1 - 2^{39}$ and 1060 and 1035 for the sequence beginning with $r_o = 0.5478126193$. The probability of random fluctuations from 1024, the expected number of such occurrences, exceeding 43, the largest of the above deviations, is 0.50 using the central limit theorem for 4096 trials. The number of occurrences of 1011 in the 7th through 10th places of the binary representation of each number was counted for each sequence, giving deviations of 4 and 18, respectively; the probabilities that such deviations would be exceeded in truly random sequences are 0.95 and 0.78 respectively.

An investigation of the density of the sequence of 4096 numbers produced from $r_o = 1 - 2^{-39}$ was made. Dividing in the interval into 200 equal sub-intervals a value of χ_o^2 of 199.41 was obtained; this is exceeded by a χ^2 variable with 199 degrees of freedom with a probability of 0.48. A different division of the same sequence into 10 equal sub-intervals gave $\chi_o^2 = 6.95$, which is exceeded by a χ^2 variable with 9 degrees of freedom with a probability of 0.58. With the same division into 10 equal sub-intervals a contingency table for r_n and r_{n+1} and for r_n and r_{n+3} producing $\chi_o^2 = 88.03$ and $\chi_o^2 = 92.33$, respectively, which are exceeded by χ^2 variable with 99 degrees of freedom with probabilities of 0.78 and 0.67 respectively. On the basis of these tests and the modest accuracy requirements in our problem this procedure was accepted as a method of generating sequences of pseudo-random numbers approximating sequences of uniformly distributed random numbers in (0,1). Furthermore, a χ^2 test was made by F. G. King on the values obtained for the probability of a blast kill in his requested 50 runs of the problem using 50 independently randomly chosen r_o's and he found the numbers acceptable for the purposes of the problem.

To produce the pseudo-random normally distributed numbers required in the random sampling procedures used in the problem an elementary and well-known device was used. If Y is a uniformly distributed random variable and if F(x) is a strictly monotone increasing cumulative distribution function whose inverse is $F^{-1}(x)$, then $X = F^{-1}(Y)$ is a random variable with cumulative distribution function F(x). Thus, for our purposes a simple approximation to the inverse of the normal distribution was used; it is based on the formula approximately inverting

$$q = \sqrt{\frac{1}{2\pi}} \int_{x(q)}^{\infty} e^{-\frac{t^2}{2}}\, dt, \quad 0 < q \leq 0.5,$$

given on sheet 67 of Form (15)s [1].

If Y is a pseudo-random number approximately a uniformly-distributed number in (0,1) and

$$Z = \sqrt{-2 \log_e \tfrac{1}{2}\,(1 - |1 - 2\,Y|)}$$

then

$$X = m + \left[Z - \frac{a_o + a_1 Z}{1 + b_1 Z + b_2 Z^2} \right] \sigma \text{ sign } (Y - .5)$$

is a pseudo-random normal variable with mean m and variance σ^2. The constants are given by

$$a_o = 2.30753 \qquad b_1 = 0.99229$$

$$a_1 = 0.27061 \qquad b_2 = 0.04481$$

This procedure of converting uniformly distributed random numbers to normally distributed random numbers requires about 80 words of storage and takes about one-third of the computation time. F. G. King has pointed out that a sum of eight, or possibly even four with a correction factor, uniformly distributed random numbers as would be a sufficiently close approximation to a normally distributed random number for the purposes of his problem. It, furthermore, would require less storage and, although more random numbers need to be generated, would require less computing time. Thus, under strait circumstances it would be advisable to use this scheme rather than the one now coded. One minor consideration is necessary; more extensive testing of the pseudo-random sequences is needed, there being several times more random numbers required.

This latter procedure for obtaining normally distributed numbers from uniformly distributed ones was coded in a later revision of the problem. It used a sum of 5 or 6 uniformly distributed numbers properly normalized and with a small correction to improve the tails of the distribution. The time saving in the running of the problem amounted to about 30% to 40% of the previous time. This is due to the elimination of the square root and logarithm routines needed in the former procedure.

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